The Development of Counting from Infancy Through the Early School Years

Humans are born with a natural ability to explore and think about quantity. In their first few years of life, through interactions with adults and their environment, children learn number words and start using this vocabulary to count and label quantities. From preschool through early elementary school, children expand their knowledge of number words. They gain fluency and flexibility in reciting the sequence of number words. They develop a deeper understanding of counting principles and how numbers in the count list represent quantities. These early number skills fuel the development of children's early arithmetic skills and prepare them for years of mathematical learning.[1–6]

In This Brief

- A summary of research findings on the development of counting knowledge and skills from infancy through the early school years
- Practical implications for early educators and caregivers working with children from birth through age eight

In this brief, we present an overview of research on the development of children's counting skills, as well as practical implications aligned with this research for educators and caregivers to support children's early math learning. These strategies were developed with teachers in mind but are also well-suited for families.

The descriptions of counting development and the associated practical implications focus on typically developing children in healthy and supportive caregiving and educational environments. However, the rate and course of individual children's counting development vary widely, and many factors can play a role in this development, such as a child's language skills[7–9] and general cognitive skills.[10–12] For example, executive function skills—such as the ability to hold information in memory, think flexibly, and regulate attention, behaviors, and thinking—may affect a child's ability to keep track when counting objects.[11] Additionally, the quality and quantity of math input and experiences in children's everyday environments play an important role.[13–15]

Additionally, research suggests that socioeconomic factors may also be associated with young children's development of counting skills.[14,16–19] Given the range of variables that contribute to children's math learning, some children may exceed the competencies that are described for a particular developmental period, while others may need more time and support to reach that level. A summary of foundations and standards in counting for infants and toddlers, preschoolers, and early elementary school students in California are presented in Appendix A.

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Sensitivity to Quantity in Infancy

Even though young infants do not know how to count, they show interest in quantities. To study this, researchers make use of an infants' tendency to look at something that is new or different for a longer period of time than something they are used to seeing. For example, researchers may show infants groups of objects (e.g., three dots) on a screen and present the same quantity in each group repeatedly, each time measuring how long the infants look at the group. After many repetitions, the researchers introduce a new quantity (e.g., two dots) on a screen. Infants' suddenly increased looking time when introduced to the new quantity suggests they notice that the quantity is different from the previous quantity. This type of research has found that very young infants are able to differentiate between quantities.

In the case of small quantities (e.g., one to three items), researchers found that seven-month-olds notice that one dot is different from two dots[20] and that two dots are different from three dots.[21] This research suggests that infants, very early on, are able to recognize the quantity of items in a set of up to three objects.[22–27]

Infants, very early on, are able to recognize the quantity of items in a set of up to three or four objects.

In the case of larger quantities (e.g., 8 or 20 items), researchers have found that six-month-olds notice that a group of 8 dots is different from an array of 16 dots[28,29] and that a puppet jumping four times is different from a puppet jumping eight times.[30] Unlike their exact differentiation of small sets, infants' early ability to tell the difference between larger quantities is approximate. Therefore, younger infants can only tell the difference between large quantities that are very dissimilar (with a ratio of about 1:2, such as 8 vs. 16).[28] However, with development, they get better at differentiating quantities that are closer together in number (with a ratio of about 2:3, such as 16 vs. 24).[28]

Infant

- Notices the difference between two small quantities (e.g., 1 vs. 2 items) precisely
- Notices the difference between two large sets that are very dissimilar in quantity (e.g., 8 vs. 16)

Toddler

- Uses a few number words but is still learning their exact meaning
- May recite small portions of the count list (e.g., 1 to 5)

Preschool

- Recites count list from 1 to 10
- Uses one-to-one correspondence to count objects
- Understands how counting can be used to determine "how many"
- Develops flexibility in counting (e.g., counts from 6 to 10)

Elementary

- Counts up to 100
- Uses skip counting by 10s, 5s, or 2s to count more efficiently
- Keeps track of counts when counting up from numbers other than 1
- Thinks about numbers as belonging to a mental numberline

Building a Vocabulary of Small Number Words in Toddlerhood

As children's language skills develop in early toddlerhood, they begin to recognize and say some number words. For example, an 18-month-old might have learned to say "one" when asked how old she is. She might have also learned to use number words to label sets that are small enough to "subitize," which means she can look at a group of one to three objects and just see how many there are without counting.[31–33] In other words, she knows what two objects looks like—just as she did when she was an infant—and she has learned that the word "two" is the correct label to use. Because toddlers learn these number words in the languages in which they hear them, dual language learners who speak a language other than English at home may at first learn to say number words in their home language.

As toddlers' language skills improve, they learn to recite small portions of the count list much in the same way they learn the alphabet, with repeated exposure and practice over time.[32] Early on, they may not hear each number in the count list as a separate word but instead hear them together as part of a string, almost like singing a song or reciting a nursery rhyme.[31,34] Around age two, they might learn to recite the count list from one to five with some errors, such as skipping or repeating a number, especially when reciting the count list beyond five (e.g., 1, 2, 3, 4, 5, 8).[31,35,36]

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Practical Implications for Adults Working with Infants and Toddlers

Infants' and toddlers' budding numerical skills can be supported by highlighting quantity and using number words when counting and labeling quantities during play and daily routines. Doing so will bring their attention to number and quantity in the world around them and will help them build numerical vocabulary that will support their emerging counting skills.

Research-Based Strategies:

- Use many opportunities in daily routines and interactions to model counting small sets of objects and refer to the quantity of the set.[14] For example, count blocks as a toddler stacks them and say how many there are altogether or point to and count pieces of fruit during snack time.
- Use hand gestures for different quantities when counting or talking about numbers in both English and the children's home language (e.g., say ¿Cuantos años tienes? ¡Dos!" and hold up two fingers).[37,38]
- Use songs or rhymes to recite portions of the count list, starting with 1 to 5, then expanding to 1 to 10.
- When reading counting books, point to and count each object on a page and say how many there are in total.[39]
- Encourage children to count in both English and their home language. Use counting books. You might want to ask families about songs, rhymes, or other practices in their home language that they enjoy and that involve counting.
- As children's language develops, encourage them to count small sets of objects and recite the count list with you. As they get the hang of it, provide opportunities to count larger sets of objects and count to higher numbers in the count list. For example, when cleaning up toys, count how many toys are going back into the toy box.

Gaining Understanding of Counting in the Preschool Years

Between the ages of three and five, children make enormous strides in their counting development. They continue to improve in reciting longer sequences of the count list, learn to count sets of objects, and come to understand that the purpose of counting is to determine "how many." As these skills come together, preschoolers develop an understanding of the meaning of the number words in their vocabulary, such as knowing that "four" always refers to four objects (or four events), but never to three objects or five objects.

Younger Preschoolers: Learning the Count List

Between the ages of three and three and a half, many children recite part or all of the count list from 1 to 10.[31,36] They may still make errors, such as skipping a number (… 4, 5, 6, 8, 9, 10), saying the same number twice (… 4, 5, 5, 6, 7, 8 …), or inverting numbers in the sequence (… 4, 5, 7, 6, 8, 9 …). Interestingly, they may make the same errors consistently,[35] which suggests that they understand the key counting principle of stable order: the order of the count list matters and should be the same in every situation.[35] Additionally, dual language learners may be more accurate at reciting the count list in one language (e.g., home language) over the other (e.g., English) depending on level of exposure and use of number words in the different languages. Even as children's ability to recite the count list improves, they might not yet understand that each word in the count list represents a quantity or know what quantity each number word represents. Still, by gaining mastery in reciting parts of the count list, they are building a strong foundation that will eventually help them develop a deeper understanding of counting.

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Core Principles of Counting

Children begin to reliably use counting to determine how many of something there is in a set when they are able to apply three core counting principles: stable order, one-to-one correspondence, and cardinality.[35] Children typically gain mastery in applying these principles during the preschool years.

- **Stable order:** Using the same sequence of number words consistently, in the same order when counting objects, regardless of whether the sequence of number words is correct
  Example. A child who consistently uses the same (incorrect) number sequence, such as "one, two, four, five, six," to count five objects, or a child who consistently says the correct sequence, "one, two, three, four, five," both demonstrate an understanding of the stable order principle.
- **One-to-one correspondence:** Assigning one and only one number word to every counted object
  Example. If given five blocks, a child applies this principle when he points to one block, saying "one," then the next, saying "two," then the next, saying "three," and so on.
- **Cardinality:** When counting objects, understanding that the last number word counted is the quantity of the set
  Example. If shown a set of five blocks and asked how many there are, the child would count the objects (1, 2, 3, 4, 5), knowing that "five" represents the total number of blocks. After initially counting the set and prompted with "How many are there?" she would say "five" without having to recount.[33,40]

Younger Preschoolers: Learning to Count Objects

Many children between the ages of three and three and a half can count a small set of objects.[33,36] As they practice counting objects, they learn to apply the key counting principle of one-to-one correspondence: one and only one number word should be assigned to each counted object.[35] When learning to apply the one-to-one correspondence principle, children at first make errors, especially when counting larger quantities.[31] For example, a child might sweep his finger over a collection of two blocks while only using one number word to count them.[36] Children of this age are better able to apply the principle of one-to-one correspondence when counting a small set of objects (e.g., five or six) and when objects are organized in a row.[31,36] They may need to touch or point to each item as they count, often moving in one direction, such as left to right or top to bottom.[31,36]

One reason for children's counting errors is that counting requires coordination of skills and knowledge that children are just beginning to master. For example, they need to coordinate saying the numbers in the count list while keeping track of the objects they are counting. Because children at this age are still developing their ability to regulate their attention and behavior,[41,42] they may make procedural errors, such as skipping an object or counting the same object twice. However, with practice and support, children gain skill in correcting procedural counting errors.[43] For example, a child may try to count a collection of six blocks too quickly, skipping one or two objects in the process. Then, when encouraged by an adult or a peer to count more slowly, count in one direction (e.g., left to right), and move objects as she counts (e.g., move all counted objects to the side), she may be able to recount the set without making errors.[31,36,43]

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Another important counting principle children have to learn is the cardinality principle: understanding that the last number word counted in a set represents the size of the set.[33,35,40] For example, when counting six blocks (e.g., "one," "two," "three," "four," "five," "six"), they know that the last word, six, represents the total number of blocks. Additionally, a child who knows the cardinal meaning of a number word such as six, understands that the word "six" refers to exactly six of something, and not five or seven.[40] The child also understands that the word "six" is not the name or a characteristic of one specific object in the set[33,44] but describes the quantity of any set of that size, whether he counts six books, six spoons, or six cars.

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Most researchers suggest that children first have to learn the cardinal meanings of smaller number words, starting with numbers one through four or five, before acquiring the cardinality principle more generally.[40,45,46] For example, many children will first learn that the word "one" refers to one object and will then learn that the word "two" refers to two objects, and so on. At this stage children may not always understand how counting can be used to figure out how many items are in a set because they are just beginning to learn the connection between counting and the quantity of objects in the set.[40]

By the time children know the meanings of the number words one through four or five (usually between ages 3.5 and 4.5), they are able to generalize what they have learned from acquiring the cardinal meanings of small number words and come to understand that counting is fundamental to determining the size of a set. Once they come to this realization, children are believed to have acquired a more generalized understanding of the cardinality principle. Now children can accurately label any quantities that fall within their counting range.[40] For example, a child who has understood the cardinality principle would be able to count seven items and identify the set size as seven. At this point in development, children also develop the ability to produce sets of objects with greater accuracy. For example, when asked "give me seven crayons," the child can count out seven from a larger set.[33,47] For dual language learners, research suggests that, if a child has understood the cardinality principle in one language, he will be able to apply this knowledge to the other language, although he may still need to learn some of the number word labels in the other language. Thus, even if a child does not know all number words in both languages, once he has learned the key counting principles, he can apply them to either language.[48]

Once children develop an understanding of the cardinality principle, they will also understand that a word's position in the count list is what determines its cardinal meaning.[40] That is, they show an understanding of the principle of succession: that each number word represents a quantity that is one more than the quantity represented by the previous number word. A child who understands this principle knows that "four" is the word you use to describe a quantity that is one more than "three."[49–51] Children

Practical Implications for Adults Working with Preschoolers—Counting Objects

Adults can support preschoolers' counting development by providing them with opportunities to practice the count list and use counting to find the total number of objects in a set. As children develop the basic counting skills, they can use counting to solve problems or answer questions (e.g., finding out which tower has more blocks).

Research-Based Strategies:

- Provide children with opportunities to practice reciting the count list in English and in their home language. For example, count while walking to the playground using songs or rhymes.
- Provide children with small collections of concrete objects that are similar in size, color, and shape. For example, invite children to count the number of leaves or rocks they have collected outside.
- Help children apply the principle of one-to-one correspondence by arranging objects in a line and moving the objects to the side as they are counted. This action can help children keep track.[31]
- After counting, restate the total amount of objects and gesture to indicate that you are referring to the whole set (e.g., by sweeping your hand around or over the set).[31]
- Invite children to produce sets of a specific quantity and encourage them to check by counting in English or their home language.[33,40,46] For example, ask children to bring five paintbrushes to the table and have them check whether they grabbed the right amount by counting them out.
- With time and practice, ask children to count items that are more abstract. For example, ask children to count the number of days until the weekend.
- Provide opportunities to use counting as a way to solve problems. For example, ask children to count how many cups are needed at their table.

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who understand the cardinality principle also get better at putting quantities of objects (e.g., collections of dots on cards) in numerical order from smallest to largest.[51] They gain skill in estimating the size of larger collections of objects (between 6 and 10), which helps them compare the sizes of numbers in the count list, such as knowing that "ten" is more than "six."[52]

As children gain a deeper understanding of the count list, they also learn to think about numbers in a more visual way, by thinking of them on a number line where smaller numbers go on the left and larger numbers go on the right.

Older Preschoolers: Building Competency in Counting

Between the ages of about four and five, children develop greater competency in counting, both in reciting the count list and in counting objects. Once they have learned to recite the count list from 1 to 10, the next hurdle is 1 to 20. Learning to verbally count to 20 is a major accomplishment, particularly in the English language, because the words that make up the count list from 11 to 19 do not clearly replicate the pattern of the count list from 1 to 9 (e.g., we do not say "ten-one" or "ten-two" but instead "eleven" and "twelve").[32,52] Once they have mastered 1–20, children typically generalize the pattern of the count list to higher decades, although it may take them some time to master the order of the decades.[53,54]

Older preschoolers also start to become more flexible counters. For example, they start to count up from numbers greater than one (such as counting from six to nine) and count backwards (such as counting down from eight to four).[31,36,54] Acquiring this flexibility is important because it helps them count objects more accurately; for example, if they make a mistake when counting objects, they might not have to start over from one, but instead can pick up from where they left off. This flexibility is a precursor to solving arithmetic problems.[55–57]

Having mastered the core counting principles (stable order, one-to-one correspondence, and cardinality), older preschoolers show a deeper understanding of numbers and quantity. For example, they are able to recognize that two sets are only equal if the items in one set can be placed in one-to-one correspondence with the items in the other set.[58] Similarly, they can identify that two sets are equal to each other even if the appearance of the two sets is very different (e.g., items in one set are spread out while items in the other set are closer together) because children at this

Practical Implications for Adults Working with Older Preschoolers—Understanding Number Relationships

Older preschoolers become much more flexible and fluent in reciting the count list and in counting objects. Once children understand the cardinality principle, they also develop an understanding of the relationship between numbers. For example, they begin to understand that six is one more than five. Educators can help children develop their flexibility and fluency in counting and have a deeper understanding of relationships between number through a variety of activities.

Research-Based Strategies:

- Provide opportunities for children to count up from numbers other than one (e.g., "Let's count from 6 to 10! 6, 7, 8, 9, 10") and to count down (e.g., Let's count down from 8 to 3! 8, 7, 6, 5, 4, 3"). For example, when standing in line to go outside, ask children to count how many are waiting in line, and ask them to count backward once they have reached the end. They may count in English or in their home language.
- Invite children to compare two quantities to identify which one is more or less. For example, during circle time, count how many boys and girls are in class and encourage children to figure out whether there are more boys or girls. Try this activity in an imaginary scenario as well, without physical items present.[52]
- Give children opportunities to practice putting quantities of objects in numerical order. For example, ask children to put collections of objects, such as stacking cubes, in numerical order from small to large amounts.[51]
- Encourage children to predict what one more than or one less than would be.[51] For example, when singing "five little monkeys jumping on the bed," use concrete objects and take away one monkey at a time, asking children to predict how many monkeys are left.

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age understand that two sets that can be placed in one-to-one correspondence with one another must have the same number of items.[59]

Developing Counting Fluency in the Early Elementary School Years

With instructional support and practice, children gain a mastery of counting and deepen their understanding of number relationships during the elementary school years. In doing so, they build a strong foundation for developing skills and knowledge in arithmetic.

Expanding Competency in Counting

Children become much more flexible and fluent in reciting the count list and in counting objects during the early elementary school years. By about age six, their count list extends to 100, and eventually 200. Around this age, they also learn ways to count more efficiently, such as skip counting by 10s, 5s, or 2s, and may even be able to count a set of objects by counting pairs within the set.[31] By about age seven they have a better understanding of the number system, recognizing that the sequence of decades (i.e., 10, 20, 30, and so on) follows the same sequence of units (i.e., 1, 2, 3, and so on).

In early elementary grades, children also become more skilled at counting up from numbers other than one and at counting backward.[31] By doing so, they are learning to keep track of the number of counts. For example, when asked, “If you had four marbles and I gave you three more, how many would you have altogether?” preschool children might need to start at one to solve the problem (a count-all strategy). Conversely, older children who have become more flexible counters might be able to start at four and count three more to seven by keeping track on three fingers, or even keeping track mentally (a count-on strategy). Thus, this flexibility enables them to use their counting skills to solve arithmetic problems more efficiently, such as counting on rather than counting all.

Number Relationships

As children gain a deeper understanding of the count list, they also learn to think about numbers in a more visual way, by thinking of them on a number line where smaller numbers go on the left and larger numbers go on the right.[60,61] For example, if you were to draw a horizontal line with “0” on the left end and “10” on the right end, a child who has developed a mental number line up to 10 would know that 5 is halfway on the number line between the endpoints.

In general, it takes time for children to build this mental number line for numbers outside of their counting range. For example, when asked to estimate the locations of numbers on a horizontal line with endpoints 0 to 100, most kindergartners will overestimate how “big” smaller numbers are and put them too far to the right on the line. On the other hand, children tend to put all larger numbers very close together toward the right side of the number line because they struggle to understand that the spacing between larger numbers (99 and 100 differ by one unit) is the same as the spacing between smaller numbers (3 and 4 also differ by one unit).[62] By the time they are in second grade, most children will estimate the locations of numbers on a 0–100 number line fairly accurately,[62] but they will show the same pattern of overestimating smaller numbers when given a number line with endpoints 0 to 1,000.[61]

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Practical Implications for Adults Working with Early Elementary Students

As children expand their counting range and acquire more flexibility in counting, they become more skilled in applying their counting skills to solving arithmetic problems. To help them build this foundation of counting skills and deepen their understanding of relationships between numbers, teachers can provide materials that highlight the structure of the number system, such as number grids, number lines, and unit blocks. Encourage children to use these materials to practice skip counting and counting flexibly (i.e., counting up or down from numbers other than one).

Research-Based Strategies:

- Provide opportunities to practice skip counting by setting up activities in which grouped objects can be counted.[31] For example, create cutouts of small baskets containing five apples each (or other groupings, such as tens, twos, or threes), and ask children to use skip counting to figure out how many apples there are in total. Encourage children to try this activity in both English and their home language.
- Use number lines or number grids to practice counting up from numbers other than one or counting backward. For example, present a number line in which units are shown with hatch marks, but only multiples of five are labeled with numerals. Ask children to start counting forward from a given number (e.g., 6) and stop at another given number (e.g., 15).
- To help children practice keeping track of the number of counts when they count on (e.g., count up 4 numbers from 7, which requires keeping track, as opposed to counting up from 7 and being told to stop at 11), use a number line or number grid, in which units are labeled with numerals, as a board game.[56]
- Highlight how the sequence of decades matches the sequence of units by referring to 10-by-10 number grids. Practice counting the decades in English and the child's home language (e.g., "diez, veinte, treinta …").
- Talk about the distance between numbers. For example, the distance between 5 and 6 is the same as the distance between 17 and 18 because both are only one number apart. Try to compare very small number pairs (e.g., <10) to very large number pairs (e.g., >50) to help children understand that two numbers that are next to each other in the count list are always the same distance apart.[62]

Conclusion

As described in this brief, counting development is rooted in an infant's ability to discriminate quantities of objects or events. As their language skills develop in toddlerhood, they build knowledge of the count list and begin to apply this knowledge to counting objects. Through such experiences, preschool children learn that counting has a stable order, that each and every object counted should have one and only one label, and that the last number counted in a set tells the quantity of the set. Building on these fundamentals, early elementary students expand their counting range and become more flexible and fluent in their counting, as they move toward a deeper understanding of numbers and the relationships between them. As competent counters, they apply these foundational skills to solving arithmetic problems.

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Appendix A: California Early Learning Foundations and Standards in Arithmetic

California Infant-Toddler Learning Foundations

Foundation: Number Sense
The developing understanding of number and quantity

8 months	18 months	36 months
At around 8 months of age, children usually focus on one object or person at a time, yet they may at times, hold two objects, one in each hand.	At around 18 months of age, children demonstrate understanding that there are different amounts of things.	At around 36 months of age, children show some understanding that numbers represent how many and demonstrate understanding of words that identify how much. (By 36 mos.; American Academy of Pediatrics 2004, 308)

Source: California Infant-Toddler Learning Foundations[63]

California Preschool Learning Foundations in Mathematics

Number Sense: Substrand 1.0

	At around 48 months of age	At around 60 months of age
Number Sense	**1.0 Children begin to understand numbers and quantities in their everyday environment.** 1.1 Recite numbers in order to ten with increasing accuracy. 1.2 Begin to recognize and name a few written numerals. 1.3 Identify, without counting, the number of objects in a collection of up to three objects (i.e., subitize). 1.4 Count up to five objects, using one-to-one correspondence (one object for each number word) with increasing accuracy. 1.5 Use the number name of the last object counted to answer the question, "How many . . . ?"	**1.0 Children expand their understanding of numbers and quantities in their everyday environment.** 1.1 Recite numbers in order to twenty with increasing accuracy. 1.2 Recognize and know the name of some written numerals. 1.3 Identify, without counting, the number of objects in a collection of up to four objects (i.e., subitize). 1.4 Count up to ten objects, using one-to-one correspondence (one object for each number word) with increasing accuracy. 1.5 Understand, when counting, that the number name of the last object counted represents the total number of objects in the group (i.e., cardinality).

Source: California Preschool Learning Foundations in Mathematics[64]

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California Common Core State Standards for Mathematics: Kindergarten–Grade 2

Domains: Counting and Cardinality; Number and Operations in Base Ten

	Kindergarten	Grade 1	Grade 2
Counting and Cardinality	K.CC **Know number names and the count sequence.** 1. Count to 100 by ones and by tens. 2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1). 3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects). **Count to tell the number of objects.** 4. Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. 5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. **Compare numbers.** 6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. 7. Compare two numbers between 1 and 10 presented as written numerals.		

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	Kindergarten	Grade 1	Grade 2
Number and Operations in Base Ten		1.NBT	2.NBT
		Extend the counting sequence.	**Understand place value.**
		1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens—called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). 2. Count within 1000; skip-count by **2s**, 5s, 10s, and 100s. **CA** 3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. 4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
		Understand place value.	
		2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones—called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). 3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	

Source: California Common Core State Standards for Mathematics[65]

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