

Diversity as a Resource Towards Students' Participation in the Mathematics Classroom

Marta Civil

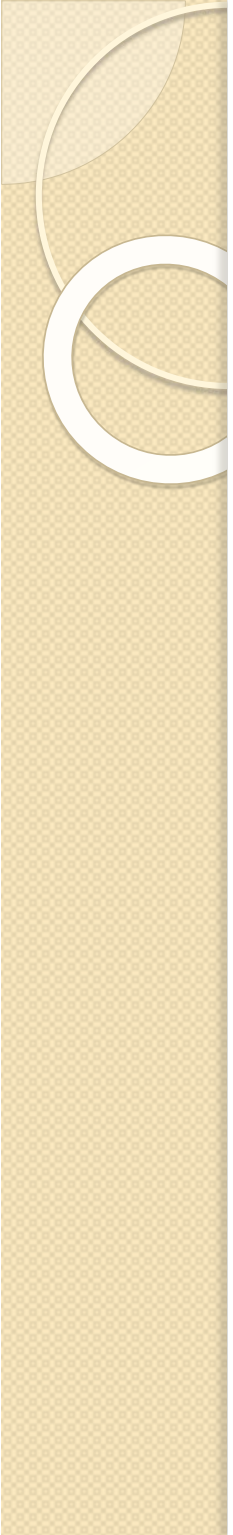
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About diversity

- Use the Chat to introduce yourselves and to share your thoughts on these questions:
 - What comes to mind when you hear the word “diversity” (in the context of mathematics teaching and learning)?
 - What does it mean to view diversity as a resource?

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- “Diversity tends to be presented as something to be accommodated, rather than an intrinsic and valuable aspect of mathematics classroom life” (Barwell, 2012, p. 324)
 - Barwell, R. (2012). Heteroglossia in multilingual mathematics classrooms. In H. Forgasz & F. Rivera (Eds.), *Towards equity in mathematics education: Gender, culture, and diversity* (pp. 315-332). Springer.

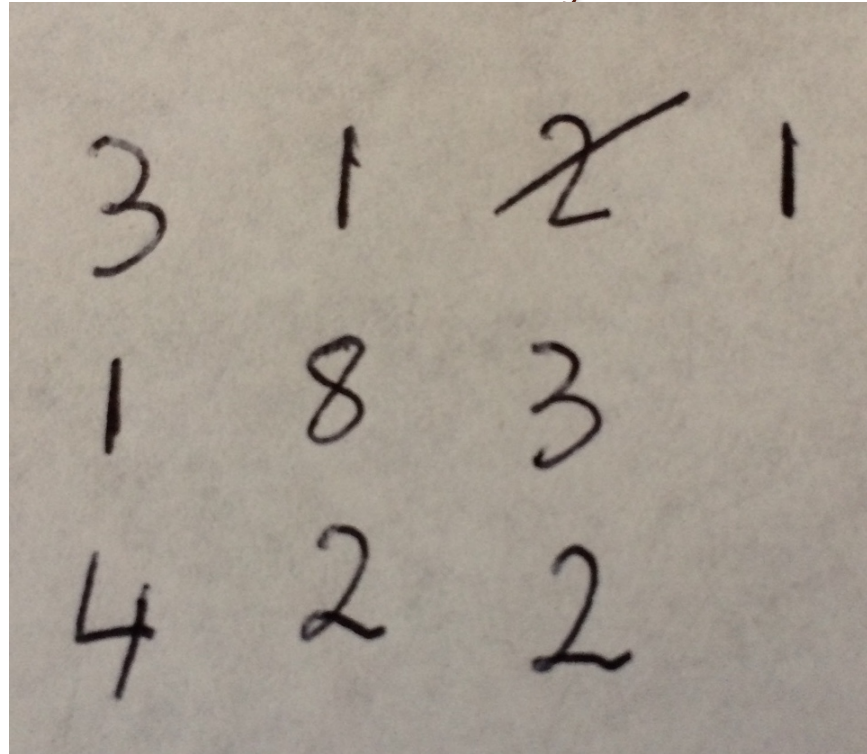
Plan for today's talk

- Exploring (cultural and linguistic) diversity as a resource towards participation
- A framework for participation
- Engaging in a task to experience some of what I am talking about

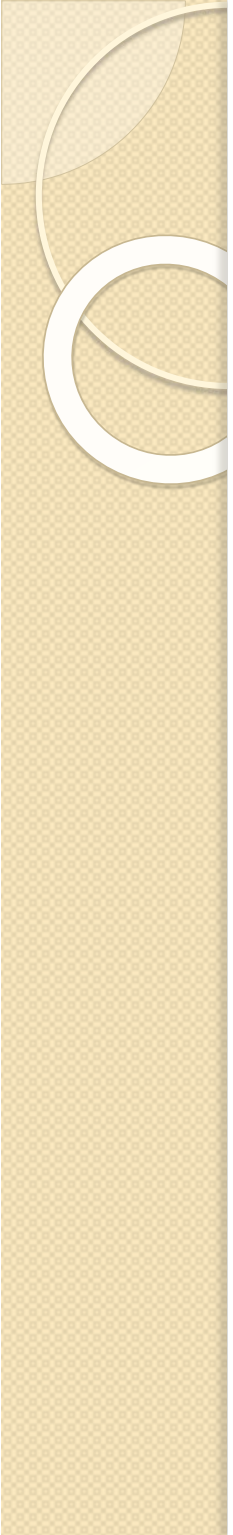


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A student shows you this...



Gorgorió, N. Planas, N. & Vilella, X. (2002), p. 38
("Immigrant children learning mathematics in mainstream schools" in G. de Abreu, A. J. Bishop, & N. Presmeg (Eds.), *Transitions between contexts of mathematical practices*. Kluwer)


- 
- How would you react? What would you do?
 - This is the way that an immigrant student had been taught how to subtract in his home country.

Different Algorithms

$$\begin{array}{r} 341 \\ -175 \\ \hline \end{array}$$

$$\begin{array}{r} 2^1 \\ \swarrow \quad 6 \\ \quad 7 \\ \quad \quad 6 \\ \rightarrow 1 \quad 6 \quad 6 \end{array}$$

“From left to right, wouldn’t kids get confused? If I showed them from left to right, I would think that when you got **to the real thing**, that they would get upset or would get confused.” (preservice elementary teacher)


$$\begin{array}{r} 73 \\ - 58 \\ \hline -5 \\ +20 \\ \hline 15 \end{array}$$


$$(3 - 8 = -5)$$

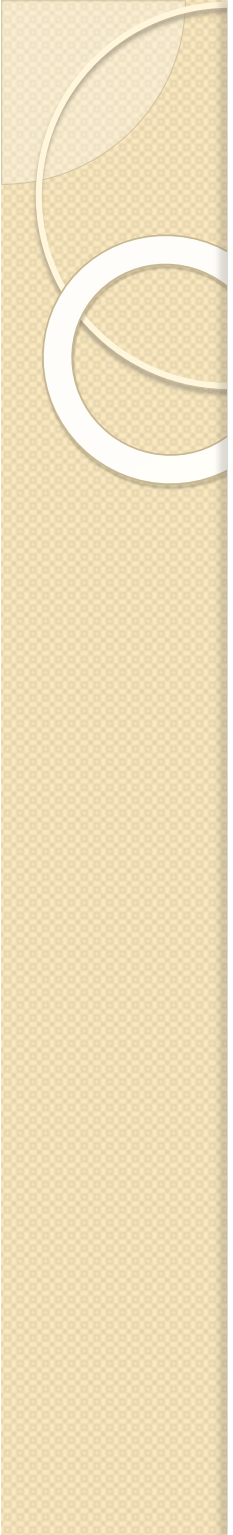
$$(70 - 50 = 20)$$

“I do believe that you could eventually convince him that learning to carry **is easier** and leaves less room for error” (preservice teacher)

How do we react to “difference”?

- The article by Perkins & Flores (2002) in *Mathematics Teaching in the Middle School*, “Mathematical notations and procedures of immigrant students” points to several differences particularly between U.S. and some Latin American countries.
- There are notable differences in how some numerals are written; the use of comma or period to indicate decimals; the use of the term billion (1 billion = 10^9 in the U.S. (1000 millions), but in many other countries, 1 billion = 10^{12} , a million millions)

- 
- And there are also several differences in algorithms (e.g., the one for subtraction earlier).
 - When preservice teachers read this article, their reactions in general were along the lines of “I didn’t know this”; “this is interesting.” But one reaction in particular caught my attention...

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- “This is nice but they need to learn to do things the U.S. way.”
 - We want to build bridges to student success.
 - Let’s look at participation more closely

A participation framework (Civil, 2014)

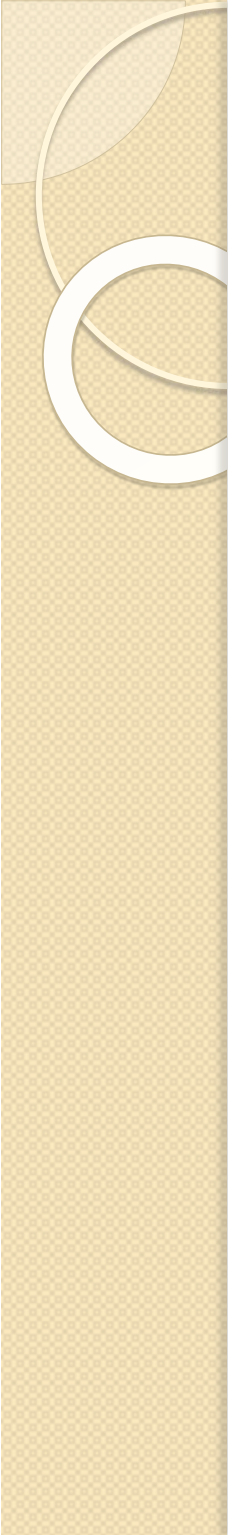
- Concept of status: What does it mean to be good at math?
- Nature of the task: Whose knowledge and experiences are represented?
- Approaches to doing mathematics: Whose and what approaches are valued?
- Language(s) in the classroom: Which language(s) and forms of communication get privileged?
- Civil, M. (2014). Musings around participation in the mathematics classroom (Guest Editorial). *The Mathematics Educator*, 23(2), 3-22.
(<https://openjournals.libs.uga.edu/tme/article/view/1737>)

1) About status

- Status is at the center of Complex Instruction (Cohen & Lotan, 1997; Featherstone, Crespo, Jilk, Parks, Oslund & Wood, 2011). I encourage you to look at this work...
- Here I'm looking at status as I first became aware of it, through the voices of fifth graders
- These fifth graders gave me a clear description of how the participation patterns operated... by status
 - Popular children (e.g., good at sports) had high status
 - Children in GATE (gifted and talented education) had high status

Cohen, E.G., & Lotan, R.A. (Eds.) (1997). *Working for equity in heterogeneous classrooms: Sociological theory in practice*. New York, NY: Teachers College Press.

Featherstone, H., Crespo, S., Jilk, L., Oslund, J., Parks, A., & Wood, M. (2011). *Smarter together! Collaboration and equity in the elementary math classroom*. Reston, VA: NCTM.

- 
- Context: a fifth-grade classroom with 29 students (18 Mexican American; 1 of Central American origin; 5 white of European origin; 4 African American; 1 Native American)
 - GATE: 7 students, 4 of them white of European origin.
 - The children are very aware:
 - Rebecca (white and in GATE): “GATE tends to be upper class white people, I’ve noticed, it’s kind of a corrupt system.”
 - Anthony (Mexican American, not in GATE): “If GATE is to make us more intelligent, how come I don’t get to be in GATE so that I can get smart?”



Let's think about these two students' comments

- They are elementary school students... what do you think your students would say about their awareness of how the education / school system works... or actually does not work?



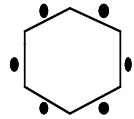
2) Nature of the task

- Whose experiences are represented in the tasks we provide students?
- Are we willing to work with different interpretations?
- What if students bring their everyday knowledge into the mathematics classroom?
- Does the real world really matter in the mathematics classroom? And whose real world?
- Let's look at one example

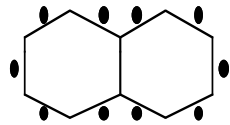
The Hexagon Problem (adapted from MARS Shell Centre Team (2009))

<https://www.map.mathshell.org/>

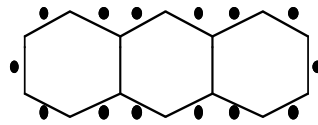
- Sarah finds how many students can sit around a row of desks.
- Complete Sarah's table (table went till 7 desks).



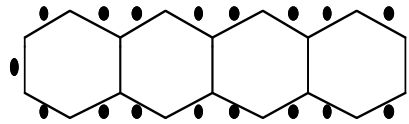
1 desk 6 students



2 desks 10 students



3 desks 14 students



4 desks

#desks in a row	# students
1	6
2	10
3	
4	
5	

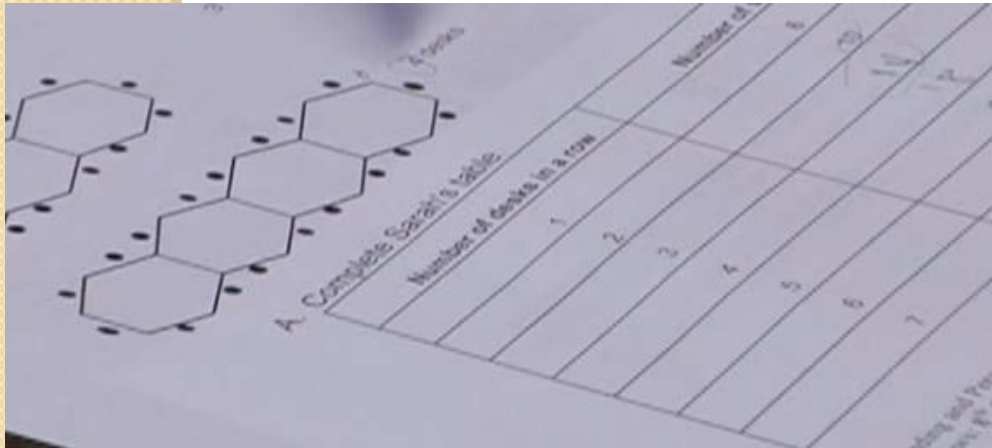
- 
- Sarah says that 47 students can sit around a row of 11 desks.

Without drawing the desks, explain whether you agree or disagree with Sarah and why.

- Octavio (8th grader) does some calculations, using the information he had from the work on the table from the previous question and has that for 11 desks it would be 46 students.

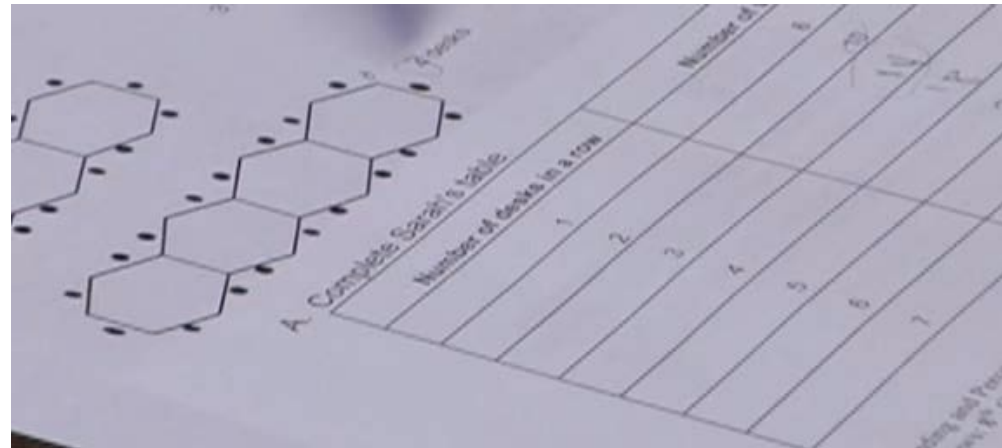
Octavio's hexagonal tables problem

- Sarah says that 47 students can sit around a row of 11 desks.



Without drawing the desks, explain whether you agree or disagree with Sarah and why.

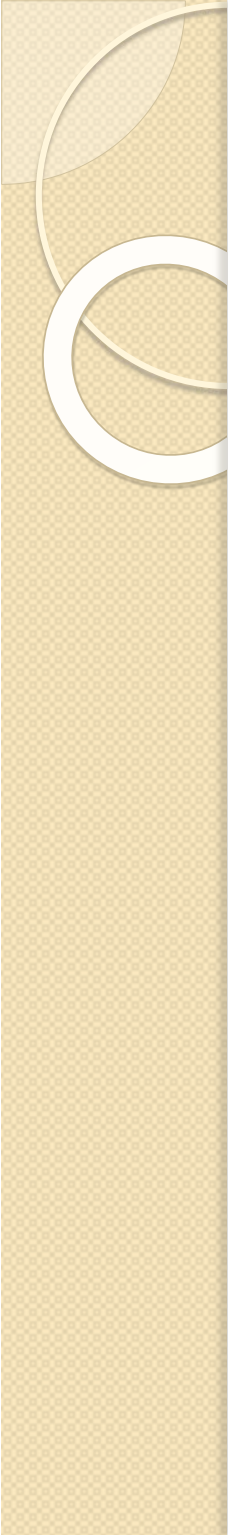
Because, if I put eight tables and it's thirty-four...
... to eleven is forty-six. Kids would be forty-six in
because it's only by one student. And it's probably ...
by putting like one student right here. [adds one dot to
a corner of the configuration]

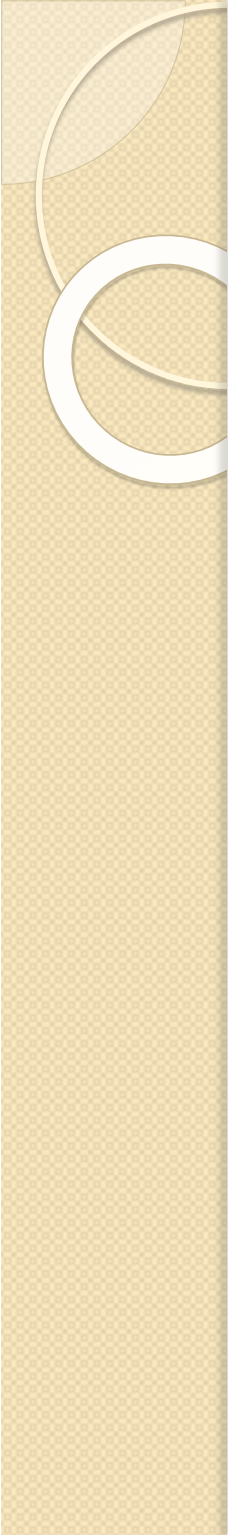


3) Approaches to doing mathematics

A watermelon and two cantaloupes cost \$4.65. Three watermelons and two cantaloupes cost \$10.15. How much does one watermelon cost?

- Could be solved algebraically (by setting up equations)
- Could be solved by looking at the problem and reasoning.
- This is what David, a preservice elementary teacher, did

- 
- David reasons that the difference between the two situations is \$5.50 which corresponds to 2 watermelons, and thus 1 watermelon costs \$2.75. However, he seems hesitant and checks his reasoning several times. Why?
 - The watermelon looks too expensive to him. (This was in the mid 80s)
 - “Actually, I probably should use algebra because I’d come with proper numbers; this [his solution] could be wrong, I’m not so confident.”

- 
- Over the years, I have noticed that students who are really trying to make sense of problems, do bring everyday experiences ... and those who don't are very good at playing the school game
 - What kinds of students (and human beings) do we want to prepare?
 - How open are we to value approaches that may be very different from our own / from the ones we teach?
 - Let me talk about valorization of knowledge...



Valorization of knowledge

- We all bring our values on what counts as mathematics; what are valid ways to do mathematics
- We saw this with the different algorithms and the reactions of some preservice teachers.
- Parents also bring valorization of knowledge.



Let's look at division

$$1224 \div 42$$

- How did you learn / were taught to divide?
- How do you divide now?
- How do you teach division?



Let's share how you were taught ...

- $1224 \div 42$

Division in Colombia, Spain, ...

$$\begin{array}{r} 1224 \\ 384 \\ 6 \end{array}$$

$$\begin{array}{r} 42 \\ \hline 29 \end{array}$$

Mexico

$$\begin{array}{r} 42 \overline{) 1224} \\ \underline{384} \\ 6 \end{array}$$

U.S.

$$\begin{array}{r} 42 \overline{) 1224} \\ \underline{- 84} \\ 384 \\ \underline{- 378} \\ 06 \end{array}$$

Scaffold method for division

$$\begin{array}{r} 29 \text{ R } 6 \\ 42 \overline{) 1224} \\ \underline{- 420} \quad \times 10 \\ 804 \\ \underline{- 420} \quad \times 10 \\ 384 \\ \underline{- 84} \quad \times 2 \\ 300 \\ \underline{- 210} \quad \times 5 \\ 90 \\ \underline{- 84} \quad \times 2 \\ 6 \end{array}$$

Mexico

$$\begin{array}{r} 42 \overline{) 1224} \\ \underline{384} \\ 6 \end{array}$$

U.S.

$$\begin{array}{r} 42 \overline{) 1224} \\ \underline{- 84} \\ 384 \\ \underline{- 378} \\ 06 \end{array}$$

A mother's reaction

- Marisol: When I looked at how he was dividing, he subtracted and subtracted and that he wrote all the equation complete I said, “this teacher wants to make things complicated. No, son, not that way! This way!” And he learned faster with this procedure [the “Mexican” way].

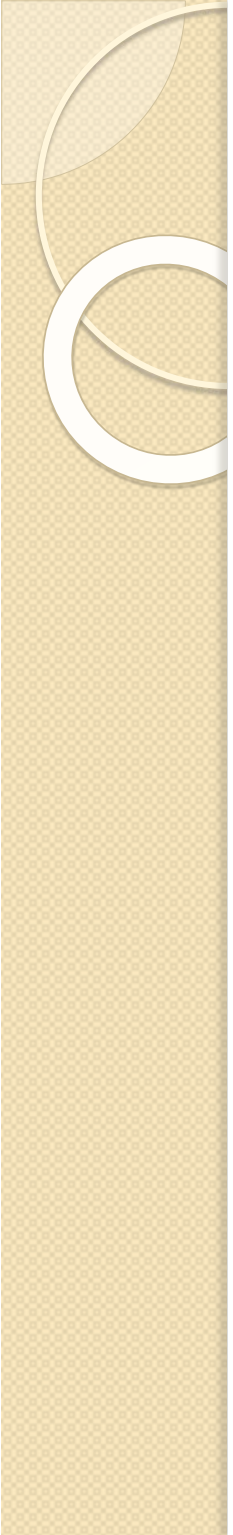
Another mother's reaction

- Verónica: I tried to do the same with my child with divisions, that he didn't write everything, but he says, "no, no, mom, the teacher is going to think that I did it on the computer." "You don't need to write the subtraction son", I say, "you only put what is left." "No, no, my teacher is going to think that I did it on the computer, I have to do it like that." "Ok, you think that..., but I want to teach you how we learned." And I did teach him, but he still uses his method, and **that way he feels safe that he is doing his homework as they told him to.** The same thing with writing above what they borrow and crossing it out, I tell him, "I remember our homework did not have to have any cross-outs," whereas his does....



Take away message

- Mathematics is not culture-free
- Think about the richness in looking at these different algorithms / methods, comparing them, seeing how they are alike / different... We can welcome this diversity, thus building bridges between students' worlds and the school world...

- 
- *“The Latino children, if their parents come from Mexico, then they probably did it a different way.... If you’re looking at algorithms, they’re going to be like “my dad does it this way” or “my mom does it this way.” And so then you’re bringing in another way so that they’re seeing maybe even a third or a fourth or a fifth way to attack a problem.” (6th grade teacher)*



Or we can dismiss it...

- “Yes, but that’s in mama’s home. Let’s do it the way that we do it in the school.” (5th grade teacher)
- What message do we send when we say, “this is the school way; that is the home way”?



4) Language & forms of communication

- Much of my work is with bilingual (multilingual) learners....
- It is about valuing languages as resources rather than as obstacles... Hence emphasis on bilingual rather than English learner.
- But it is also about students whose home language is English but may have patterns of interaction or use forms of English that are not what the school / teacher is used to.
- For example, students of certain cultural backgrounds may find it difficult to engage in mathematical arguments if that involves challenging peers' ideas or the teacher's (e.g., work in NZ by Hunter and colleagues).



Some questions to ponder

- What are the implications of restrictive language policies for the learning of mathematics?
- What are we missing by not having / providing access to multiple languages in the classroom?
- What do we gain by having students bring in multiple resources (languages; humor; everyday experiences)



Experiencing some of this...

- Let's look at the nature of the task and the language component... you are now traveling to Catalonia... And we are going to work on a problem set in a cultural context unique to this part of the world.

Els Castells

L'estructura general d'un castell és la pinya, el tronc i el pom de dalt. Els castells s'anomenen segons el número de persones que hi ha a cada pis del tronc i el número de pisos. Per exemple, podem tenir un 2 de 8, 3 de 8 o 4 de 8. Pel problema que us proposem, prenem el castell de 4 de 8. Tenint en compte que el pom de dalt està format pels dosos, l'acotxador i l'enxaneta (2+1+1), sabríeu trobar el número de faixes que necessitem per les persones del tronc i del pom de dalt d'aquest castell?



Use the chat

- To put your thoughts on what this problem is about...

Això és un castell



L'estructura general d'un castell és la pinya, el tronc i el pom de dalt. Els castells s'anomenen segons el número de persones que hi ha a cada pis del tronc i el número de pisos. Per exemple, podem tenir un 2 de 8, 3 de 8 o 4 de 8. Pel problema que us proposem, prenem el castell de 4 de 8. Tenint en compte que el pom de dalt està format pels dosos, l'acotxador i l'enxaneta ($2+1+1$), sabríeu trobar el número de faixes que necessitem per les persones del tronc i del pom de dalt d'aquest castell?

Castell 3 de 8

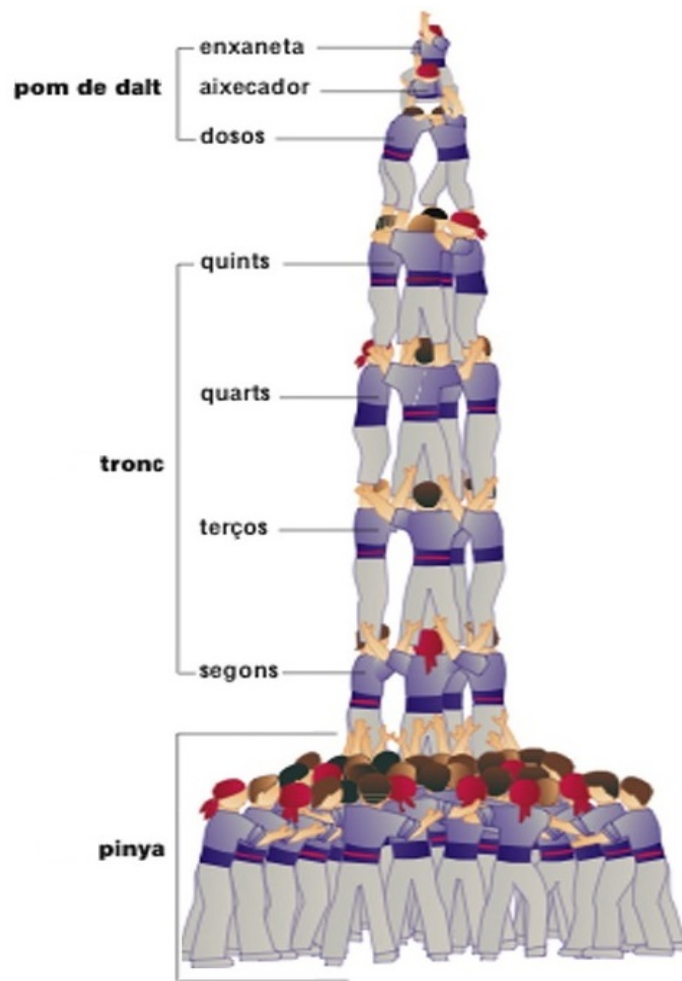




[https://www.youtube.com/watch?v=K1H
WyUIZ5kk](https://www.youtube.com/watch?v=K1HWyUIZ5kk)

[this is a longer version of the clip I
showed]

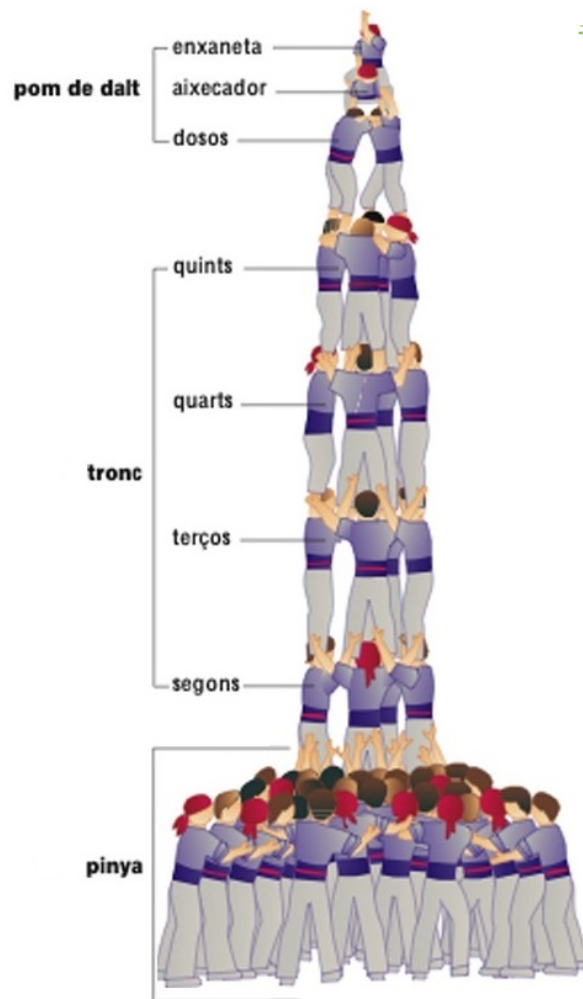
Does this help you solve the problem?



- L'estructura general d'un castell és la pinya, el tronc i el pom de dalt. Els castells s'anomenen segons el número de persones que hi ha a cada pis del tronc i el número de pisos. Per exemple, podem tenir un 2 de 8, 3 de 8 o 4 de 8. Pel problema que us proposem, prenem el castell de 4 de 8. Tenint en compte que el pom de dalt està format pels dosos, l'acotxador i l'enxaneta ($2+1+1$), sabríeu trobar el número de faixes que necessitem per les persones del tronc i del pom de dalt d'aquest castell?

4 de 8 – Xiquets de Tarragona:

https://www.youtube.com/watch?v=30BELZWc_bE

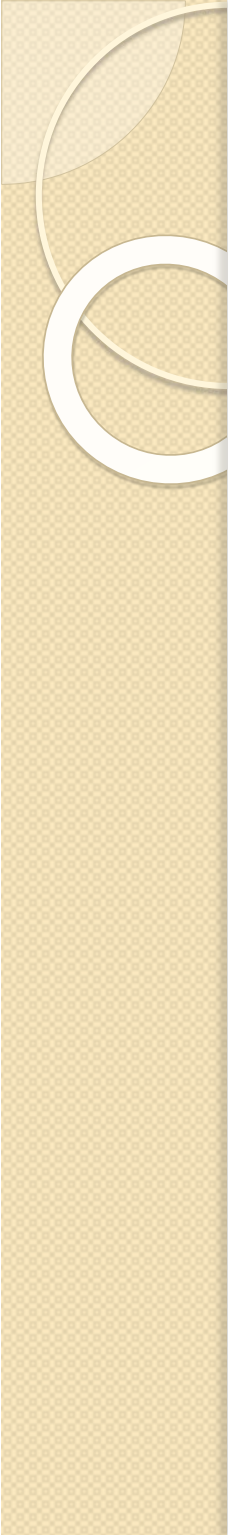


- Resolent el problema: número de faixes que necessitem per les persones del tronc i del pom de dalt d'aquest castell
- Al tronc: $4 \times 4 \rightarrow 16$ persones
- Al pom de dalt: $2 + 1 + 1 \rightarrow 4$ persones
- En total: $16 + 4 \rightarrow 20$ faixes



So....

- How did you feel during this experience?
- Use the chat to share your reactions



I chose this activity intentionally because it not only stresses what it may look like to learn / do mathematics in a language other than your home language(s), but also within a cultural context that may be very significant to the groups represented in the problem but quite foreign to others.


Language matters

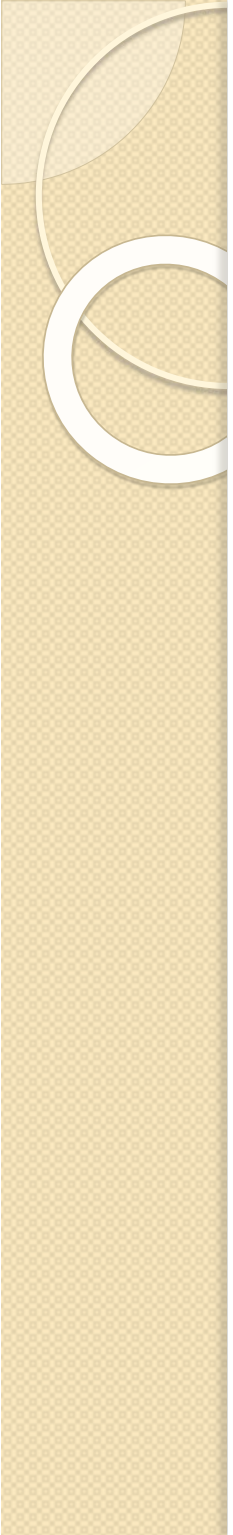
Context matters

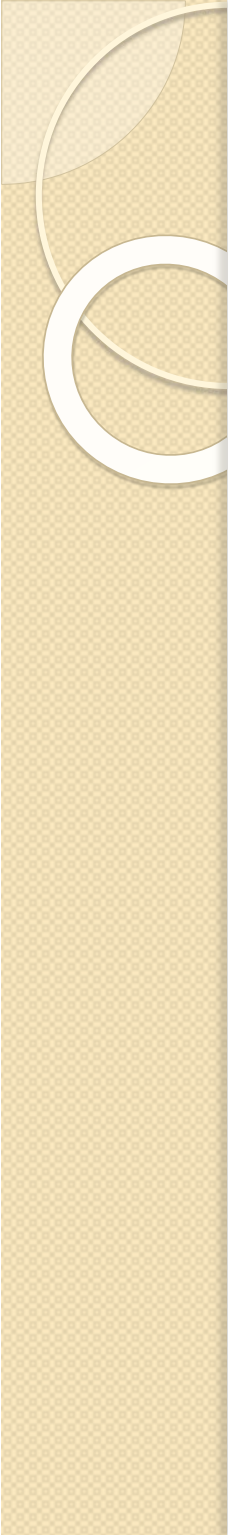


Getting to know your community

- I argue for the need to know and understand more deeply students' communities and for changing some of the ways we may go about doing things, if needed.
- What do you know about your students? Their families? Their activities outside the school day?
- Here are some statements that I often hear when working with teachers in the communities I do my work...

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- “The problem is the language... they don’t speak English”
 - Actually they speak more than 1 language; cognitive advantages of bilingualism
 - “Parents don’t care... Education is not that important for them”
 - I have yet to meet a parent who does not care about their children’s education
 - Lack of motivation
 - Children are very engaged and motivated in their community
 - Lack of parental involvement
 - What is our view of involvement? Physical presence in the school?

- 
- We need to move away from deficit language and deficit views on non-dominant students and their families
- Focus on the many strengths they bring to school




What do we do when things don't look like what we are used to?

- Students and their families DO bring (or know) different approaches in mathematics.
- As teachers, what kind of mathematical knowledge and dispositions do we need to address these different approaches?
- It is not “just” about knowing (or finding out) what mathematical knowledge students bring to school but about what to do with that knowledge.



In closing...why is attention to diversity important?

- It has always been important (or should have been), but now even more..
- As we teach differently, with more group work, discussions, etc...
- What are the implications of an emphasis on discourse for the participation of all students?
- Who has a voice? (status in the class)
- How are different contributions viewed and valued? (bringing in everyday experiences when “not appropriate”)

- 
- I want to argue for a mathematics teaching and learning that values and promotes difference...
 - But I'm afraid that instead we often lean towards a “pedagogy of control... a strategy of regimentation that responds well to the schools’ historical mission to erase or stifle diversity by creating uniformity, as deemed necessary for national (elite) interests and unity” (Moll, 2004, p. 126).

[Moll, L. C. (2004). Rethinking resistance. *Anthropology & Education Quarterly*, 35(1), 126-131.]

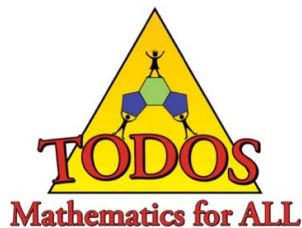


Why do I do this work?

When those who have power to name and to socially construct reality, choose not to see you or hear you, whether you are dark-skinned, old, disabled, female, or speak with a different accent or dialect than theirs, when someone with the authority of a teacher, say, describes the world and you are not in it, there is a moment of psychic disequilibrium as **if you looked into a mirror and saw nothing** (Adrienne Rich, 1986, p. 199)

[In Cultural Citizenship and Educational Democracy Author(s): Renato Rosaldo Source: Cultural Anthropology, Vol. 9, No. 3, Further Inflections: Toward Ethnographies of the Future (Aug., 1994), pp. 402-411]

- I hope this presentation has given you some ideas on how to go about doing some of this. For me a place to start is getting to know the students' communities through them and their parents. Being a learner as much as a teacher has been the most gratifying experience...
- As a place to start I suggest you take a look at the resources that TODOS: Mathematics for All offers, for example their position statements: <https://www.todos-math.org/statements>



<https://www.todos-math.org/todos-2021-conference>

2021 Virtual Conference

Activating Agency, Access, and
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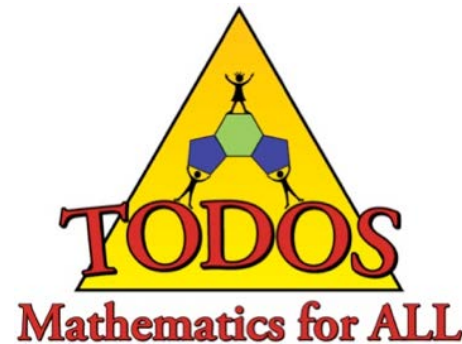
June 21-23, 2021


<https://www.todos-math.org/>

2021 Virtual Leadership Institute

Change-Agents Taking Action
For Equity In Mathematics
Education

June 24-25, 2021





Thank you!
Comments / questions